Appendix E
Theory of Combining Flow at Open Channel Junctions (Confluences)

E-1. General

In the design of flood-control channels, one of the more important hydraulic problems is the analysis of the flow conditions at open channel junctions. The junction problem is common in flood- control channel design as flows from the smaller drainage basins generally combine with those in larger main channels. The momentum equation design approach has been verified for small angles by Taylor (1944) and Webber and Greated (1966).[1] The US Army Engineer District (USAED), Los Angeles (1947), developed equations, based on the momentum principle, for the analysis of several types of open channel junctions commonly used in flood-control channel systems. Model tests of several confluence structures with various conditions of flow have been made, and the experimental results substantiated those calculated theoretically by the equations. This appendix is a presentation of the detailed derivation of the momentum equation.

E-2. Theory and Assumptions, Tranquil Flow

a. Plate E-1 gives a definition sketch of a junction. The following assumptions are made for combining tranquil flows:

(1) The side channel cross section is the same shape as the main channel cross section.

(2) The bottom slopes are equal for the main channel and the side channel.

(3) Flows are parallel to the channel walls immediately above and below the junction.

(4) The depths are equal immediately above the junction in both the side and main channels.

(5) The velocity is uniform over the cross sections immediately above and below the junction.

[1] References cited in this appendix are included in Appendix A.

Assumption (3) implies that hydrostatic pressure distributions can be assumed, and assumption (5) suggests that the momentum correction factors are equal to each other at the reference sections.

b. The use of the momentum equation in the analysis of flow problems is discussed in detail on page 49 of Chow (1959). Plate E-1c shows the forces acting on the control volume through the junction. The net force acting in the direction of the main channel is given by

$$F_{1-3} = P_1 + P_2 \cos \theta + W \sin \alpha - P_f - P_3 - U \tag{E-1}$$

where

P_1, P_2, P_3 = hydrostatic pressure forces acting on the control volume at the reference sections

$P = \gamma b y^2/2$ for rectangular section

γ = specific weight of water (62.5 pcf)

b = width

y = depth

θ = angle of intersection of the junction

W = weight of the water in the control volume

α = angle of the channel slope ($\tan \alpha$ = channel slope

P_f = total external force of frictional resistance along the wetted surface

U = unknown reaction force exerted by the walls of the lateral in the upstream direction

The change in momentum per unit of time in the control volume is equal to the net force acting on the control volume (Newton's Second Law of Motion). The change in momentum in the direction of the main channel is

$$F_{1\text{-}3} = \frac{\gamma}{g}\, Q_3 V_3 - \frac{\gamma}{g}\, Q_1 V_1 - \frac{\gamma}{g}\, Q_2 V_2 \cos\theta \qquad \text{(E-2)}$$

where

g = acceleration due to gravity = 32.2 ft/sec^2

V_1, V_2, V_3 = average channel velocity at the reference sections

Q_1, Q_2, Q_3 = discharge of the appropriate channels

When Equations E-1 and E-2 are equated, the basic momentum equation for the flow through the junction is obtained.

E-3. Simplification of General Equation, Rectangular Channels

a. If the slope is appreciable, the evaluation of the hydrostatic pressure distributions will involve a correction factor $\cos^2 \alpha$ (Chow 1959). However, for slopes normally employed for flood- control channels, this correction factor will be negligible. For slopes less than 10 percent ($\alpha \approx 6$ deg) the $\cos \alpha$ and $\cos^2 \alpha$ terms can be neglected in the momentum equation and the result will be accurate to within 1 percent.

b. The unknown reaction force U has been assumed by Taylor (1944) and Webber and Greated (1966) to be equal and opposite to the pressure term from the lateral; that is,

$$U + P_2 \cos\theta \qquad \text{(E-3)}$$

and the pressure term from the lateral is balanced by the pressure force on the curve wall *BC* in Plate E-1a. This assumption is reasonable as long as the depth in the region of the curved wall (area *ABC* in Plate E-1a) is basically uniform and the curvature of the streamlines is not appreciable.

c. The component weight of fluid acting along the main channel is equal to the frictional resistance for both uniform flow and gradually varied flow; that is,

$$P_f = W \sin\alpha \qquad \text{(E-4)}$$

can be assumed as long as the flow is not rapidly varying. This is the basic assumption of uniform flow; i.e., the total force of resistance is equal to the gravitational force component causing the flow. Introducing those three simplifications, the momentum equation reduces to

$$P_1 - P_3 = \frac{\gamma}{g}\, Q_3 V_3 - \frac{\gamma}{g}\, Q_1 V_1 - \frac{\gamma}{g}\, Q_2 V_2 \cos\theta \qquad \text{(E-5)}$$

d. Introduction of the hydrostatic pressure distribution in Equation E-5 leads to the following:

$$\frac{\gamma b y_1^2}{2} - \frac{\gamma b y_3^2}{2} - \frac{\gamma}{g}\,(Q_3 V_3 - Q_1 V_1 - Q_2 V_2 \cos\theta) \qquad \text{(E-6)}$$

By the use of the continuity equation at each reference section

$$Q_1 = A_1 V_1;\; Q_2 = A_2 V_2;\; Q_3 = A_3 V_3 \qquad \text{(E-7)}$$

where A is the area. Dividing by the unit weight of water, the equation can be simplified to

$$\frac{Q_3^2}{gA_3} + \frac{b y_3^2}{2} = \frac{Q_1^2}{gA_1} + \frac{Q_2^2}{gA_2} \cos\theta + \frac{b y_1^2}{2} \qquad \text{(E-8)}$$

If a further assumption is made that the side channel width is equal to the main channel width, this equation can be generalized. The papers by Taylor (1944) and Webber and Greated (1966) contain the details of the derivation including graphs of the equation and experimental data.

E-4. Unequal Width of Main Channels

The derivation of the momentum equation for a rectangular channel with unequal widths follows very closely that outlined in the preceding paragraphs. Plate E-1b gives a definition sketch for this type of junction. The only additional force is that pressure force not balanced by the curved wall *DC*. The pressure ΔP_1 is the component in the main channel direction of the hydrostatic pressure acting over the width *EF* at reference section 2. The effective width for computing ΔP_1 is $(b_3 - b_1)$ and the pressure is

$$\Delta P_1 = \gamma \left(\frac{b_3 - b_1}{2} \right) y^2 \qquad \text{(E-9)}$$

By adding appropriate subscripts and omitting γ for simplicity, the momentum equation then becomes:

$$\frac{Q_3^2}{gA_3} + \frac{b_3 y_3^2}{2} = \frac{Q_1^2}{gA_1} + \frac{Q_2^2}{gA_2} \cos\theta + \frac{b_1 y_1^2}{2} + \frac{(b_3 - b_1)}{2} y_1^2 \qquad \text{(E-10)}$$

This can be further simplified to

$$\frac{Q_3^2}{gA_3} + \frac{b_3 y_3^2}{2} = \frac{Q_1^2}{gA_1} + \frac{Q_2^2}{gA_2} \cos\theta + \frac{b_3 y_1^2}{2} \qquad \text{(E-11)}$$

E-5. Trapezoidal Channels

a. The hydrostatic pressure distribution in a trapezoidal cross section is given by

$$P = A\bar{y} = y^2 \left(\frac{b}{2} + \frac{Zy}{3} \right) \qquad \text{(E-12)}$$

where

$\bar{y}$ = distance of the centroid of the water area below the surface of the flow

y = flow depth

b = bottom width of the trapezoidal cross section

Z = side slope, horizontal to vertical

Introduction of this term with the proper subscripts in the basic momentum equation will give the following:

$$\frac{Q_3^2}{gA_3} + \left(\frac{b_3}{2} + \frac{Zy_3}{3} \right) y_3^2 = \frac{Q_1^2}{gA_1} + \frac{Q_2^2}{gA_2} \cos\theta + \left(\frac{b_1}{2} + \frac{Zy_1}{3} \right) y_1^2 \qquad \text{(E-13)}$$

b. The equation for unequal widths of trapezoidal channels is derived in much the same manner as for unequal width of rectangular channels given in paragraph E-4. The inclusion of the hydrostatic pressure distribution terms for a trapezoidal cross section in that equation will result in

$$\frac{Q_3^2}{gA_3} + \left(\frac{b_3}{2} + \frac{Zy_3}{3} \right) y_3^2 = \frac{Q_1^2}{gA_1} + \frac{Q_2^2}{gA_2} \cos\theta + \left(\frac{b_3}{2} + \frac{Zy_1}{3} \right) y_1^2 \qquad \text{(E-14)}$$

E-6. Energy Loss

The energy loss at a junction H_L can be obtained by writing an energy balance equation between the entering and exiting flow from the junction.

$$H_L = E_1 + E_2 - E_3 \qquad \text{(E-15)}$$

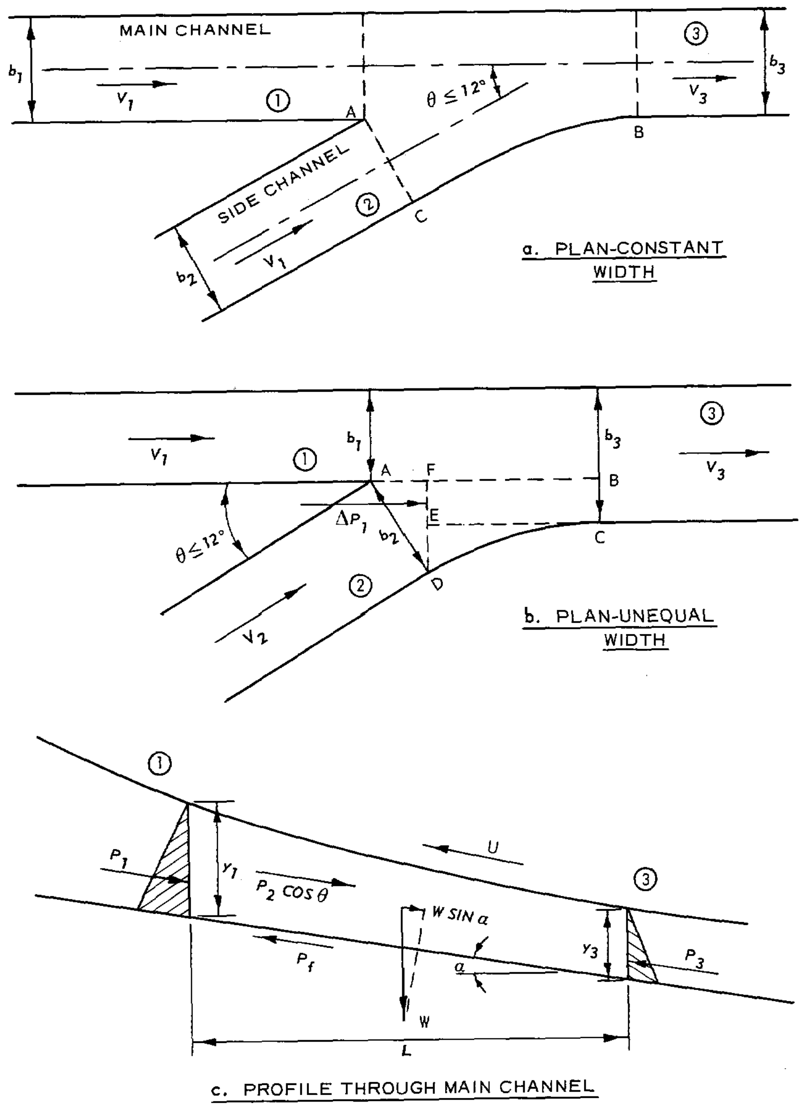
The momentum and continuity equations could be used to obtain depths and velocities for evaluating the specific energy at the sections. However, it is not desirable to generalize the energy equation because of the many types of junctions.

E-7. Rapid Flow

In contrast with tranquil flows at junctions, rapid flows with changes in boundary alignments are generally complicated by standing waves (Ippen 1951). In tranquil flow, backwater effects are propagated upstream, thereby tending to equalize the flow depths in the main and side channels. However, backwater cannot be propagated upstream in rapid flow, and flow depths in the main and side channels cannot generally be expected to be equal. Junctions for rapid flows and very small junction angles are designed assuming equal water-surface elevations in the side and main channels (paragraph 4-4d(1)(a)). Model tests by the USAED, Los Angeles (1949), on rapid-flow junctions have verified the use of the momentum equation developed in this appendix for this purpose.

E-8. Sample Computation

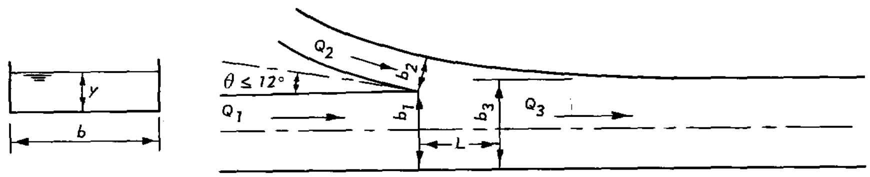
Typical momentum computations for a confluence are given in Plate E-2. The computation conditions are for the type of junction developed by the USAED, Los Angeles, to minimize standing wave effects.



RECTANGULAR CHANNEL JUNCTION

DEFINITION SKETCHES

PLATE E-1



GIVEN DATA

DISCHARGES	CHANNEL WIDTHS	FLOW DEPTHS	FROUDE NO.	ANGLE & LENGTH
Q_1 37,000 CFS	b_1 110 FT	y_1 12.29 FT	F_1 1.39	θ = 0°
Q_2 5,000 CFS	b_2 36 FT	y_2 12.29 FT	F_2 0.57	$\cos\theta$ = 1
Q_3 42,000 CFS	b_3 145 FT	y_3 10.40 FT	F_3 1.52	L = 100.0 FT

MOMENTUM EQUATION *

$$\frac{Q_1^2}{gA_1} + \frac{Q_2^2}{gA_2}\cos\theta + \frac{b_1 y_1^2}{2} + \frac{(b_3 - b_1)}{2} y_1^2 = \frac{Q_3^2}{gA_3} + \frac{b_3 y_3^2}{2} \qquad \text{(VI-4)}$$

$y_{c_3} = 13.80$ $\qquad$ $0.85\, y_{c_3} = 11.73$

MOMENTUM UPSTREAM*	FT³	MOMENTUM DOWNSTREAM	FT³
$\frac{Q_1^2}{gA_1} = \frac{(37{,}000)^2}{32.2 \times 1{,}351.9}$	= 31,449	y_3, ESTIMATE = 10.40	
$\frac{Q_2^2 \cos\theta}{gA_2} = \frac{(5{,}000)^2 \times 1}{32.2 \times 442.44}$	= 1,755	$\frac{Q_3^2}{gA_3} = \frac{(42{,}000)^2}{32.2 \times 1{,}508.0}$	= 36,328
$\frac{b_1 y_1^2}{2} = \frac{110 \times (12.29)^2}{2}$	= 8,307	$\frac{b_3 y_3^2}{2} = \frac{145 \times (10.40)^2}{2}$	= 7,842
$\frac{(b_3 - b_1)}{2} y_1^2 = \frac{(145 - 110) \times (12.29)^2}{2}$	= 2,643		
ΣM_{1-2}	= 44,154	ΣM_3	= 44,170

* THE TERM γ (SPECIFIC WEIGHT OF WATER) HAS BEEN OMITTED FROM ALL TERMS OF THIS EQUATION AND THE FOLLOWING COMPUTATIONS.

RECTANGULAR CHANNEL
JUNCTION
COMPUTATIONS

PLATE E-2